PRESSURE OF GRANULAR MATERIAL ON WALL OF MODEL SILO*

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Experimental static and dynamic pressures acting on the wall of a model silo are reviewed. The experimental static wall pressures are in a good agreement with the results calculated according to the Janssen's and Reimbert brothers' method. But the experimental dynamic overpressures and their distribution on the wall of a silo differ from those calculated by the present design standards. The pulsating character of these dynamic overpressures has been demonstrated experimentally.

The problems concerning the stress and gravity flow of granular material in bunkers and silos have been of interest especially in the last twenty years as construction of high-capacity storage units expands. Theimer[1], Paterson[2] and Garg[3] have discussed the design procedures for calculation of the wall stress of granular material which is the decisive factor for design of the wall thickness of bunkers and silos and they have pointed to considerable deviations in their application. They have also indicated the lack of reliable experimental data on the wall stress of granular material in silos especially during their discharge.

The basis for calculation of the stress of static granular material in a silo has been paved by Janssen[4] who has considered mathematically the static equilibria of vertical and shear forces acting on a differential horizontal slice defined in a bed of material. This calculation has been later verified experimentally and has become the basis for standardized calculations of wall thickness of bunkers and reinforced concrete silos for storage of grain in Germany[5], Switzerland[6], the U.S.S.R.[7] and elsewhere. A number of authors[8-15] present summaries of measurements performed with an effort to verify the theories of Janssen and they confirm a good agreement of the calculated and experimental static wall pressures. But the mentioned measurements have resulted in new conclusions according to which the stress of granular material substantially increases at the gravity flow of granular material inside the silo.

The reason of these overpressures is considered to be either the repeated breakdown of local arching patterns[16] of granular material, the mechanism of which is identical with the acoustic shock wave effects[17], or in the change of the wall friction factor at the discharge of material from the bunker[18]. Most thoroughly are theoretically developed considerations concerning the effect of internal flow pattern in a silo on the wall stress of moving material[19]. The wall stresses in bunkers during their filling and discharge have been reviewed by Turitzin[20]. He describes two types of flow of granular material inside silos. The dynamic flow is characterized by a motion of the whole material storaged which is accompanied by a dynamic increase of pressure.

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The non-dynamic flow without the pressure increase during the discharge appears when there exists the flowing core of material whose dynamic pressure effects are dumped by the bed of stationary material near the wall so that the dynamic pressures are not expected to appear in this case. Deutsch and Clyde[21] describe the internal flow pattern of granular material at its discharge from the model silo. They present relation between the fundamental flow patterns of material and the existence of overpressures. They propose an approximate upper bound plasticity solution for the pressures on the walls of silos. Jenike[22] solves equilibrium equations of granular material under the assumption of existence of radial stress field in the hopper. The solution is the slope of the hopper which may be determined by use of a graph if the angle of wall friction and the effective internal angle of friction according to Jenike of the storaged material are known. The design procedure should result in the motion of granular material without arching during discharge of storage units.

In the theory of bin wall loads Jenike and Johanson[23] proposed that three loading conditions should be considered in the analysis of loads acting on a bin wall. Initial loading which occurs when bulk solid is charged without any of it being withdrawn, flow loading which occurs after flow has been established and switch loading which occurs during the switch from initial to flow loading. The last one is transient but while it acts, it exerts large concentrated wall forces.

The mechanism of origin of dynamic pressures and the gravity flow of granular material inside the silo is studied by other authors[24-26]. Platonov and Ivanov[27] have derived an equation characterizing the mechanism of stress transfer within the flowing granular material which is analogous to the transfer of loads in the arch. Dubynin and Kramadžan[28-30] have as well applied the arching effect for explanation of origin of dynamic overpressures. They present empirical relations for the magnitude of amplitude and frequency of these pressures. They consider the dynamic overpressures as the main reason for appearance of cracks in the walls of silos and they suggest the circumferential discharge hole in the silo or a discharge consisting of several smaller orifices.

It is necessary to mention here the role of the standardized design procedures[5-7] for calculation of the wall load of storage units. They are based on the equation by Janssen[4] which includes the empirical corrective constants or constants based on the speculative assumptions which are, with respect to the discharge, increasing the value of the calculated wall pressure.

But these procedures have been proved unsatisfactory and their practical application is frequently unsuccessful (cracks in the walls of silos, collapse of the whole storage units).

EXPERIMENTAL

Model silo. Pressure measurements of granular material on the wall of an experimental silo have been made in a model silo of circular cross section — see Fig. 1. The model silo consists of six mutually interchangeable cylindrical steel sections. In one of these sections (measuring) was situated the pressure cell for measurement of pressure σ_{Hw} and of the friction component τ_{w} of wall stress of granular material. One central discharge orifice (ID 20 mm) was situated in the flat bottom and the granular material was filled by a central inlet pipe.

The apparatus has been equipped with a filling and discharge hopper enabling measurement of wall stresses at steady flow of material in the model silo. The vertical transport of granular material between the discharge and filling hopper was pneumatical.

For simultaneous measurement of the normal and shear wall stresses of granular material a device[31,32] designed by us has been used — see Fig. 1. Basic part of this measuring device is a steel dynamometric ring **1** on which the stress of granular material acts. The acting force is taken by the sensor **2** situated in the wall **3** of the silo. The bending stresses in dynamometric ring 1 are measured by the electric resistance strain gauges R_1 — R_4. The pressure and friction effects of granular material which are simultaneously acting on the ring of the measuring device are separated by use of two different wirings of tensometric strain gauges in the Wheatstone bridge. The temperature compensation is included in the wiring of the electric strain gauges. The measured pressures are registered.

Characteristics of granular material. The granular material with which the wall stresses were measured was silica sand. The mean static bulk density γ^{ST} was determined by weighing the material present in a known volume of the model silo. This volume was understood to be the total volume of material except of stationary material at the flat bottom which formed the conical discharge cone. The mean bulk density of flowing granular material γ^{FLOW} was determined in a similar way, the weighed mass of granular material was the mass which was flowing through the silo under conditions of steady flow at the constant level of material. The steady flow was considered to exist when the volume of material in the experimental silo had been five times exchanged. The values of $\gamma^{ST} = 1\,651\ \text{kp/m}^3$ and $\gamma^{FLOW} = 1\,474\ \text{kp/m}^3$ were obtained.

For determination of the angle of internal Φ_i and external (wall) friction Φ_w, the classical box shear cell method was used. For measurement of the wall friction factor f_w the steel plate was used whose degree of surface working was identical with that of the internal surface of the model silo. The values $\Phi_i = 31°$ and $\Phi_w \approx 33°$ were obtained. The second result, quite unusual in shear tests, was compared with the results of static wall stress measurements in the model silo. This is because the coefficient of wall friction f_w can be also determined during the measurement of stress as the ratio of shear τ_w and horizontal stresses σ_{H_w} on the wall. In Fig. 2 are com-

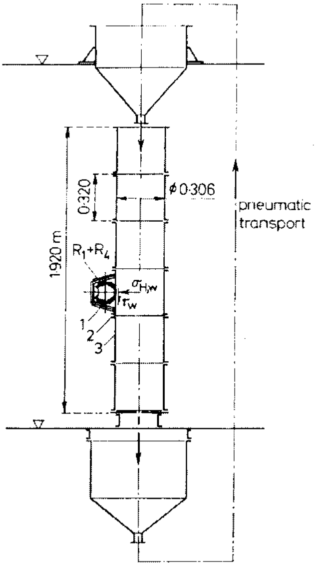

FIG. 1
Experimental Model Silo

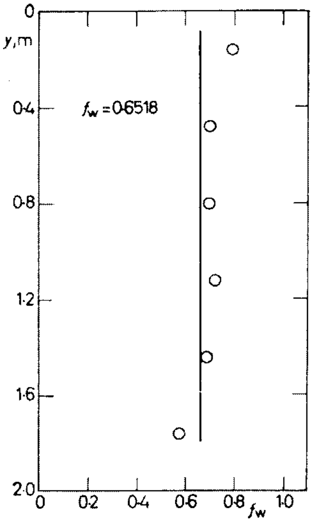

FIG. 2
Distribution of Wall Friction Factor

pared values of the wall friction factor f_w evaluated on basis of measurements of static wall stress and the constant wall friction factor obtained by the classical shear test. The greater value of the wall friction factor in comparison to the internal friction has thus been confirmed by an independent measurement. Its value is dependent on the degree of working the surface and on the feed rate (coarse feed) at machining of the internal wall surface of the model silo. The wall friction factor was also increased by the sharp-edged sand which was selected as the experimental granular material. The mutual relation between the internal and external (wall) friction $\Phi_i < \Phi_w$ was confirmed by a repeated measurement. This experimental conclusion is now being studied theoretically. For calculation of wall pressures the friction factors used have been obtained at measurements of the wall stress in the model silo (see Fig. 2).

The particle size distribution is expressed graphically (Fig. 3) by the modified exponential relation by Rosin, Rammler and Sperling in the form

$$\log (\log 100/R) = n \log x + C , \qquad (1)$$

where

$$C = \log (\log e) - n \log \bar{x} . \qquad (2)$$

The dimension of the mean particle size $\bar{x}$ is given by the total screen residue $R = 100/e = 36{\cdot}79\%$. As is demonstrated in Fig. 3 the granular material has not contained the fraction of sizes $x = 1{\cdot}80$ to $2{\cdot}25$ mm, which had been separated in advance for other purposes.

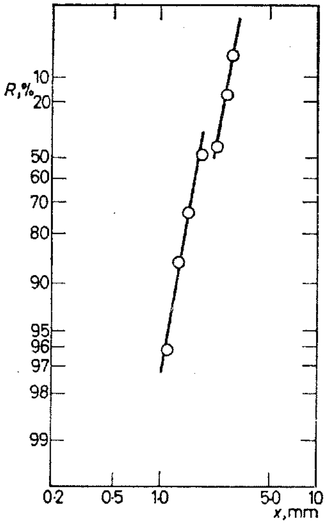

FIG. 3

Particle Size Distribution of Sand

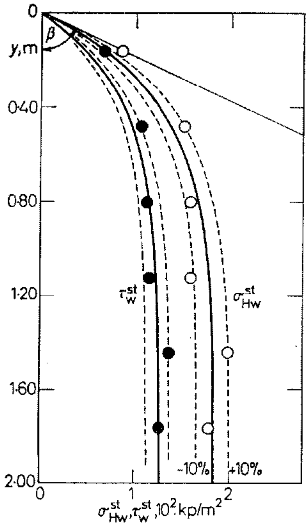

FIG. 4

Comparison of Experimental Results of Static Wall Stress with the Calculation According to Janssen[4] (solid lines)

RESULTS

Static wall pressures. The measurement of the static wall stress of granular material in the model silo has begun simultaneously with the filling of the silo. As the rate of filling and the used method of filling through the central pipe were in all made experiments the same, their effect on the measured stress could not have been determined. The long run measurements had not revealed any effect of settling of the material on the wall stress. It seems that this fact known from industrial application could not have been observed in our model silo due to the small amount of material storaged and thus due to the small pressure deformations inside this material. The measurement has been carried out in various depths from the free surface of granular material.

The characteristic measurement of static pressure σ_{Hw}^{ST} and of shear stress τ_w^{ST} of sand on the wall of the model silo is demonstrated in Fig. 4. The experimental static pressures in the model silo are in a good agreement with the pressure calculations according to Janssen where for the ratio of pressures k is used the relation

$$k = \sigma_{Hw}^{ST}/\sigma_{Vw}^{ST} = (1 - \sin \Phi_i)/(1 + \sin \Phi_i)\,, \qquad (3)$$

even though for our case of rough wall surface is more frequently recommended the equation

$$k = (1 - \sin^2 \Phi_i)/(1 + \sin^2 \Phi_i)\,. \qquad (4)$$

By analysing the Janssen's relation for calculation of the wall pressure the value of angle β can be determined from Fig. 4, according to which

$$\operatorname{tg} \beta = d\sigma_{Hw}^{ST}/dy = k\gamma^{ST}\,. \qquad (5)$$

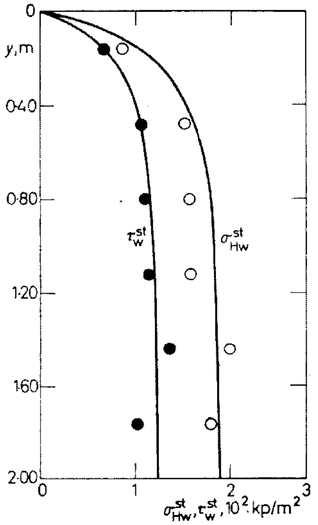

FIG. 5

Comparison of Experimental Wall Static Pressures with the Calculation According to Reimbert brothers[10] (solid lines)

The experimental results of static stress are compared in Fig. 5 with the calculation of the silo wall load according to the Reimbert brothers' method[10]. It may be concluded that, as concerns the distribution of stress the results of calculation by Reimberts are in the upper part of the silo closer to the experimentally determined pressures. The classical method by Janssen gives in the upper part of the silo slightly lower values of static load. But the experimental results of wall stress of storaged material do not differ by more than ±10% of nominal value from those calculated.

Dynamic wall pressures. Simultaneously with measurements of wall stress of sand during discharge, the flow pattern inside the silo was taken into consideration. The measurements were performed in the model silo of I.D. 306 mm in which Novosad and Surapati[33] have studied the kinematics of sand in gravity flow. The characteristic

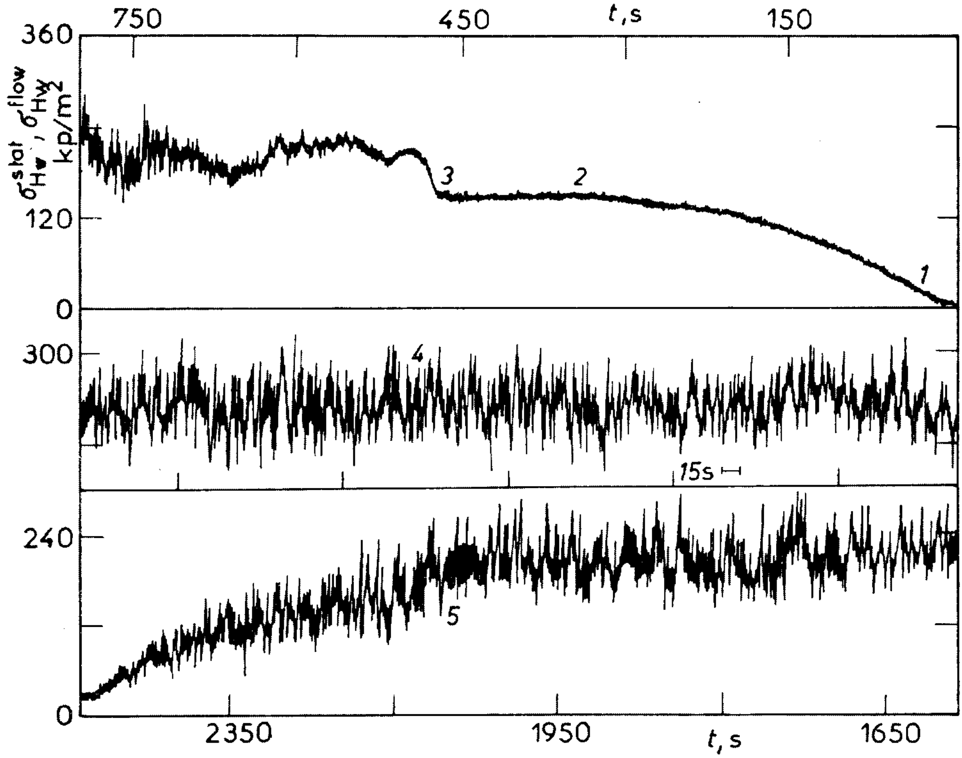

FIG. 6

Time Record of Wall Stresses of Granular Material in the Experimental Silo

Chart speed 40 mm/min, flow rate 0·005 m^3/min. *1* Filling start, *2* filling stop, *3* flow start, *4* steady flow, *5* discharge.

flow pattern inside the silo obtained in the cited paper by the method of paraffinic castings proves the existence of a flowing core. The motion of material inside the silo is thus of the type: first in-last out *i.e.* the funnel-flow.

Prior to each measurement of the wall stress the calibration check was made of the pressure measuring device. The level of material was constant in all measurements and was equal to the upper edge of the silo wall. For measurements of the wall stress of the flowing sand, the material level was kept constant and maximum by use of the filling hopper situated above the silo.

Simultaneously with opening of the central discharge orifice there appears a substantial increase in the recording of the normal and shear wall stresses which, with the development of the flow region within the silo, increases (Fig. 6). Together with the increased stress in this starting phase of motion of granular material it is also possible to observe a change in the record of the measured stress. It becomes of pulsating character and after steadying the flow it oscillates between the maximum and minimum values. The time necessary for steadying of the flow of granular material in the silo has not been studied in detail. But we have observed that the flow is steady after one exchange of material in the silo.

As expected, the minimum value of wall stress measured at flow of granular

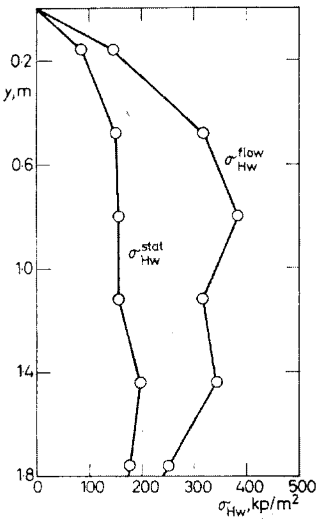

FIG. 7

Dynamic Wall Pressure Distribution of Granular Material in the Model Silo

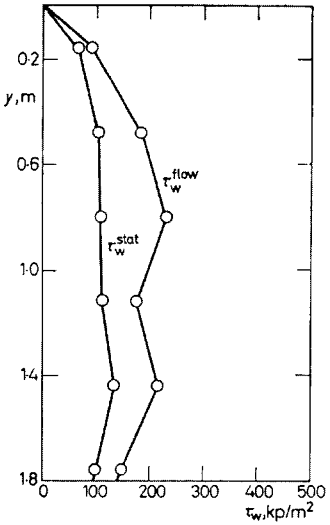

FIG. 8

Distribution of Dynamic Shear Stress on the Wall of Model Silo material was only slightly greater than the value of static stress corresponding to the given bed height. The maximum value of the measured wall stress which was the main object of our interest was in dependence on the height of location of the pressure measuring device in the wall of the silo given in Figs 7 and 8. As long as the measurements of wall stress of granular material in the full-scale reinforced silos and steel bunkers prove to have a similar character it is necessary to take into account that the wall of silo or bunker is loaded dynamically during motion of the material *e.g.* at its discharge. In such case it is of course necessary to take into account the fatigue of the construction material.

We have not studied the effect of size and location of the discharge orifice in the flat bottom of the silo or the effect of the rate of discharge (flow) of granular material on the wall stress. But we expect that the rate of gravity flow of material in silos is not affecting the magnitude of wall stress of granular material but that it may result in the change of frequency of the observable stress oscillations.

Design Procedures and Experimental Results

Only selected design methods most widely applied are used for comparison with the results of our experiments. For measured pressure values are considered the maximum

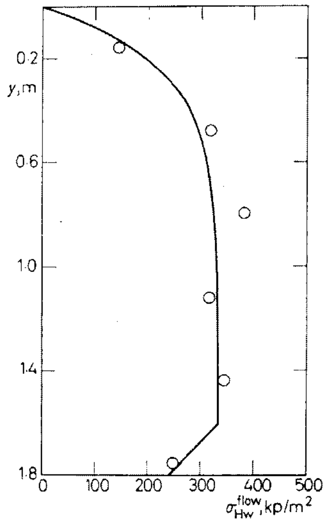

FIG. 9

Comparison of Wall Stress of Granular Material During Flow with the Calculation According to DIN 1055[5]

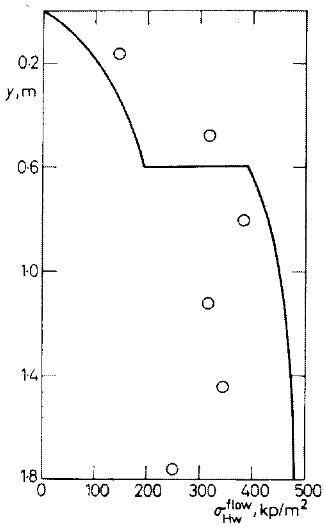

FIG. 10

Comparison of Wall Stress of Granular Material During Flow with the Calculation According to CH-302-65[7] pressure pulsations. The minimum pressures are close (slightly greater) to static pressures.

Din 1055, *Blatt* 6 (*Germany*)[5]. As is obvious from Fig. 9, our experimental wall stresses in the model silo are greater by 18% at the most than the stresses calculated according to the German Standard, DIN 1055 (ref.[5]). This difference has been observed in the central part of the height of the model silo. In the lower one third of the experimental silo the agreement of the calculated results with the experimental wall stresses of granular material is good.

Soviet Code CH-302-65 (*USSR*)[7]. The design values of pressure of flowing material on the wall of the experimental silo according to the above quoted Soviet Code[7] are quite different as compared with the experimental pressures – see Fig. 10, both in magnitude and in their distribution along the height of the silo. This is obviously due to the principle of the method which is based on the Janssen's calculation of static pressure of granular material without considering various states and properties of the storaged material during its movement within the silo. In the upper one third of the height of the silo the calculated pressures by the cited method have reached

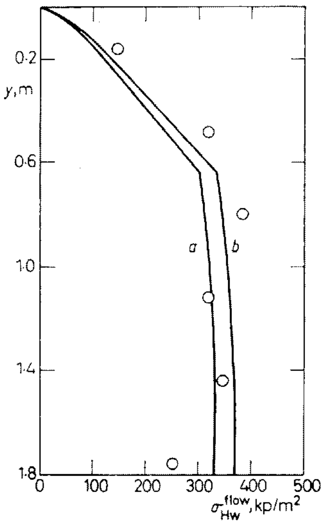

FIG. 11

Comparison of Wall Stress of Granular Material During Flow with the Calculation According to Safarian[3,4]

a for $\gamma^{FLOW} = 1\,474$ kp/m^3, b for $\gamma^{STAT} = \gamma^{BULK} = 1\,651$ kp/m^3.

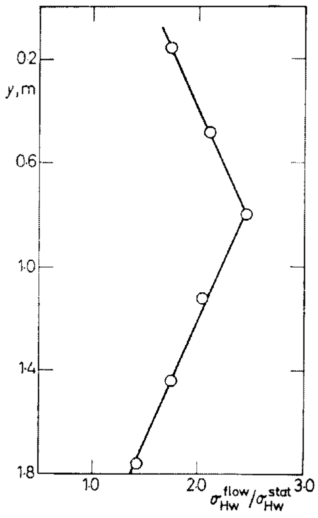

FIG. 12.

Distribution of Dynamic Ratio of Wall Stress $\sigma_{Hw}^{FLOW}/\sigma_{Hw}^{STAT}$ in the Experimental Silo only 55% of the experimental pressures. On the contrary, in the lower two thirds of the height of the silo this method given higher pressures than those experimentally determined. This unbalanced design procedure would result in a costly design of silos. In this case, for calculation of pressures was considered beside the dynamic factor also the recommended loading factor $m = 1\cdot30$.

Pressure Scheme According to S. S. Safarian (USA)[34]. Comparison of wall stresses of granular material during flow with the calculation according to the cited pressure scheme is given in Fig. 11. The agreement is good in the lower half of the height of the silo. In the upper half of the height are the calculated pressures slightly lower in comparison with those experimentally determined.

ACI *Committee* 313 (*USA*). The report of ACI Committee 313 (ref.[35]) is recommending for calculation of the wall load of silos the Janssen's equation. With respect to the new evidence on dynamic wall pressures of the storaged granular material at the discharge of silos this recommendation appears to be conservative, and underestimates the importance of dynamic pressures of the moving storaged material.

It may be concluded that some of the above given calculation methods chosen for verification of our experimental data are giving lower stresses. Only the calculation according to the DIN 1055 and the Safarian's pressure scheme are close to the experimental values. This supports the – at present very frequent – opinion that the presently used methods of calculation of wall pressures do not give an accurate basis for the design of storage units.

Our measurements of stresses of flowing granular material in the experimental silo give, as compared to the values of static pressures, considerably greater wall stresses. Distribution of this overpressure ratio along the wall given in Fig. 12, is characteristic for measurements presented in this paper.

The interesting results we have obtained in our experiments is the pulsating character of wall stresses of the flowing granular material. The measurements have proved its existence under the conditions of the so-called nondynamic flow *i.e.* at the existence of the flowing core of granular material as it had taken place in our experimental silo. On basis of pulsing character of wall pressure the conclusion can be made that at the discharge of the model silo the slip-stick flow occurs.

It has not been possible to discuss here all the so far published methods of calculation of the wall stresses of granular material moving in the silo. First of all design methods assuming linear (hydrostatic) distribution of wall stresses have not been taken into consideration due to the disagreement with results of our experimental data. Next, the methods based on the so-called mass flow of granular material (Jenike and Johanson[11], Walker[24]) were not evaluated. These methods cannot be applied in our experimental silo and are more convenient for hoppers. Moreover, the comparison has not been made of our experimental results with those methods (*e.g.*

Platonov and Kovtun[36]) in which empirical constants are used for calculation of wall stresses of granular material in motion.

CONCLUSIONS

Experimental measurements of wall stresses of granular material in a model silo for which a pressure cell designed by us has been used:

a) have confirmed validity of theoretical views on magnitude and distribution of wall stresses of stationary granular material storaged in a silo; *b*) have determined wall stresses and their distribution of the selected granular material in the model silo along the silo wall under the steady gravity flow of granular material; *c*) have demonstrated that the load of the silo wall under gravity flow of granular material is dynamic with a pulsating character. They have proved the existence of this effect in the experimental silo at the so-called nondynamic flow *i.e.* at the existence of the flowing core of granular material which demonstrates the mutual dependence of pressure pulsations and slip-stick flow model; *d*) have proved that the existing design methods for calculation of wall load of bunkers and silos do not represent either the magnitude, or the distribution or the character of the actual wall load of the model silo by the flowing granular material.

The discussed standardized design methods provide in majority of cases lower values of wall stresses than has been experimentally proved and none of the methods is considering the dynamic (pulsating) load of the silo wall by granular material under gravity flow.

LIST OF SYMBOLS

C	constant
D	internal diameter of silo
e	base of natural logarithm
f_i	internal friction factor
f_w	wall friction factor
k	stress ratio
m	load factor
R	total screen residue
x	particle size
$\bar{x}$	size of mean statistical particle
y	distance from the top of material in silo
γ^{ST}	bulk density of static granular material
γ^{FLOW}	bulk density of flowing granular material
σ_H	horizontal pressure
σ_V	vertical pressure
τ_w	wall shear stress
Φ_i	angle of internal friction
Φ_w	angle of wall friction

Subscripts

H	horizontal
V	vertical
w	wall

Superscripts

STAT	static granular material
FLOW	flowing granular material

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